Wallner lines revisited

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Wallner lines, named after the scientist who explained them, constitute a unique "postmortem" tool for evaluating crack velocity in brittle materials. These lines are delicate tracks observed on fractured surfaces of brittle materials induced by the interaction of the crack front and transverse acoustic waves generated either naturally by discontinuities such as reflections at sample edges or artificially by a transducer. Here we offer a simple accurate method to analyze Wallner lines. This method is based on first principles and yields in addition to crack velocity, also the velocity in which the information about crack initiation moved up to the Wallner line origin. In addition it brings forth the points of origin of the fracture and the Wallner line. © *2006 American Institute of Physics*.
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Wallner lines are continuous undulations appearing on flat crack surfaces, notably the "mirror plane," but they occasionally occur in the "mist zone" as well. They appear as a result of an interaction between transverse acoustic waves, generated either naturally by flaws which often occur along the rim of the fractured material or artificially by a transducer (see, e.g., Refs. 1 and 2). They are named after the scientist who explained their origin[3] and have been used since as a means to evaluate the crack front velocity v_{cr} or rather the ratio of this velocity to the transverse wave speed c_s, v_{cr}/c_s (denoted here by u). The interaction of the crack front and the wave induces locally a mode III perturbation of the driving elastic field of the front,[4] causing it to curve. This curvature along the interaction points appears as a line—the Wallner line. Recently[4] Wallner lines were invoked as a possible explanation for the "crack front waves" detected by several authors (e.g., Ref. 5). The delicate Wallner lines have little significance for studies of crack velocity of ceramics and rocks due to their masking by grain boundaries, while they are very useful in investigations of cracking of quartz single crystals, glass and glass ceramics, diamonds, and tungsten.

The three geometrical methods used previously for Wallner line analysis[6,7] [given by Eqs. (1)–(3)] are shown in Fig. 1:

$$u = \frac{\cos\alpha}{\cos\beta}, \tag{1}$$

$$u = \frac{\sin\varphi}{\sqrt{\cos^2\beta_1 + \cos^2\beta_2 + 2\cos^2\beta_1\cos^2\beta_2\cos^2\varphi}}, \tag{2}$$

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$$u = \frac{\sin(\varphi/2)}{\cos\beta}, \tag{3}$$

where the angles for Eqs. (1)–(3) are shown in Figs. 1(a)–1(c), respectively, and presently discussed. All of these methods use measurements of angles: the angle between the direction of crack propagations and the Wallner line (α), between two Wallner lines (φ), or between lines connecting two Wallner line origin points, the point of their intersection,

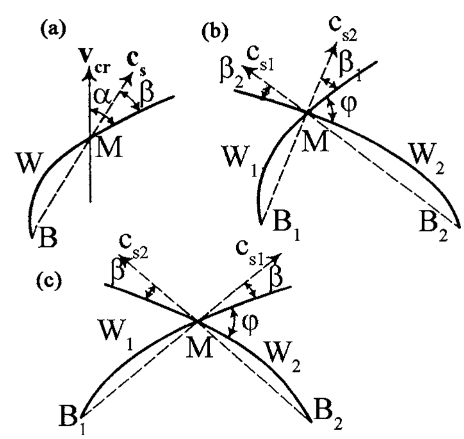

FIG. 1. Three (schematic) previous methods of crack velocity evaluation on the basis of Wallner line analysis. (a) The crack propagates upwards (black arrow) with a velocity v_{cr} while the Wallner line, depicted by W, originates from point B. They intersect, e.g., at point M, to which crack velocity calculation by Eq. (1) is related. The angle α is the angle between the crack propagation direction and the Wallner line, while β is the angle between the Wallner line and the line connecting points B and M. c_s is the transverse acoustic wave speed. (b) Asymmetrical intersection of two Wallner lines originated at points B_1 and B_2. Crack velocity evaluation here is carried out by Eq. (2). β_1 is the angle between the first Wallner line (depicted by W_1) and the line connecting its origin B_1 and point M. β_2 is the angle between the second Wallner line (depicted by W_2 and originated at point B_2) and line B_2M. φ is the angle between the two Wallner lines. (c) Symmetrical intersection of two Wallner lines. All marks are analogous to (b). Crack evaluation here proceeds by Eq. (3).

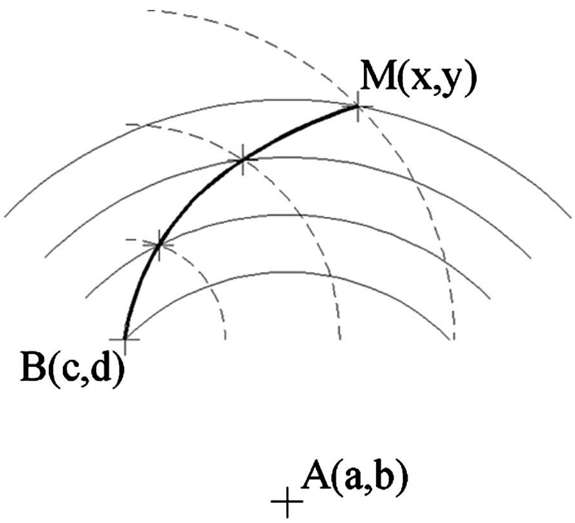

FIG. 2. Wallner line creation and the variables and parameters for its analysis. The fracture starts to propagate at $t=0$ from A and moves outwards with velocity v_{cr}. Circles (full lines) concentric to A mark stroboscopic pictures of the propagating crack front. The acoustic wave origin is at B. It starts to send waves at time $t_1(>0)$ when the information from A has reached it. The acoustic waves emanated from B move with a velocity c_s and are depicted by the stroboscopic concentric circles (dashed lines) around B. The intersections (e.g., M) of the crack front and the acoustic waves lead to the creation of the Wallner line (bold curve in the figure).

and the Wallner lines themselves $(\beta, \beta_1, \beta_2)$. For the implementation of all three methods a precise knowledge of at least two important data is needed: either (1) the crack nucleation position (and/or crack propagation direction) and (2) the point of origin of at least one Wallner line; or, alternatively, the points of origin of two Wallner lines. A precise definition of crack propagation direction/origin is possible only when there exist additional fracture marks on the crack surface (note that even if a groove is present, wherefrom the crack had started, the latter's exact position is only inaccurately defined). The location of the point of origin of the Wallner line is also usually known only imprecisely, especially when the crack propagates in an intensive manner. As a result and due to the relatively large errors incurred in measuring angles, crack velocities obtained by Wallner line analysis are often grossly inaccurate or even unobtainable.

Here we propose a method of Wallner line analysis, which can remedy these problems, and in the process, produce additional valuable information. Thus, in addition to the evaluation of the crack velocity magnitude, the points of origin of both the crack and the Wallner line are obtained, as well as the velocity by which the information about the fracture initiation traveled from the latter's origin up to the origin of the Wallner line.

The calculation is as follows (Fig. 2). The interaction, at time t, of the crack front and the acoustic wave, propagating from A (at time $t=0$) and from B (at time t_1 when information reached it), respectively, is given by

$$(x-a)^2 + (y-b)^2 = v_{cr}^2 t^2, \tag{4}$$

$$(x-c)^2 + (y-d)^2 = c_s^2 (t-t_1)^2, \tag{5}$$

where x and y are the current coordinates of Wallner line point M, A (a,b) is the crack initiation point, B (c,d) is the origin of the acoustic wave, v_{cr} is the crack velocity, and c_s is the transverse wave speed.

Eliminating t from Eqs. (4) and (5) leads to Eq. (6), an implicit relation describing the Wallner line contour $y(x)$ through

$$z \equiv (x-c)^2 + (y-d)^2 - w[\sqrt{(x-a)^2 + (y-b)^2} - s]^2 = 0, \tag{6}$$

where $w(=1/u^2)=c_s^2/v_{cr}^2$ and $s=v_{cr}t_1$ is the distance the crack propagates in the time interval t_1. Equation (6) is used together with measured points across the Wallner line to extract the unknown parameters a, b, c, d, w, and s by a least squares method.

The distance from the crack origin to the origin of the Wallner line is now given by $r=\sqrt{(c-a)^2+(d-b)^2}$. Hence the velocity of the information about the crack, v_{inf}, is obtained by

$$\frac{v_{inf}}{c_s} = \frac{r}{c_s t_1} = \frac{r v_{cr}}{c_s s} = \frac{ru}{s} = \frac{r}{s\sqrt{w}}. \tag{7}$$

We conducted our numeral experiments by using the nonlinear-fit package of MATHEMATICA that enabled us to fit the six parameters of the implicit function z [Eq. (6)]. Crack photographs were zoomed out, contours of Wallner lines were carefully mapped, and x and y coordinates of their points were collected in data files together with z values (zero).

To check the validity of our method, we next considered three known examples (see Fig. 3) where Wallner line analysis proved successful in the past for relatively accurate evaluations of crack velocities and show that crack velocity values obtained by our method are indeed similar to the previously obtained ones. In addition we discuss the supplementary information derived by our method.

Figure 3(a) shows an example of a fracture surface in inorganic glass tested under uniaxial tension.[8] A white circle marks the location of the crack origin[8] while the location of the Wallner line origin is unknown. Denoting this location by B we have calculated the crack velocity ratio u, the coordinates of the crack and Wallner line (BP) origins, and v_{inf}. The crack velocity ratio u at this point was calculated in Ref. 7 by the method depicted in Fig. 1(b) to be 0.26, while our method yielded the value of 0.264. The white arrows in Fig. 3(a) show our calculated points of the crack origin (obtained indeed close to the white spot) and the Wallner line origin B. The distance s that the crack propagates during the delay time t_1 (the time between the crack and Wallner line origins) is 0.57 mm while the distance between the crack and Wallner line origins is 1.93 mm. It means that the information here traveled at a velocity whose ratio to the transverse acoustic one is [by Eq. (7)] (1.93/0.57)0.26=0.88. This result is quite intriguing. It implies that the information moves with a velocity similar to the Rayleigh one. This observation is currently under investigation.

The second calculation was conducted with the two Wallner lines marked by AB and BC in Fig. 3(b) formed by the interaction of the crack front and an artificial shear wave from a special transducer.[1] Our calculated crack velocity val-

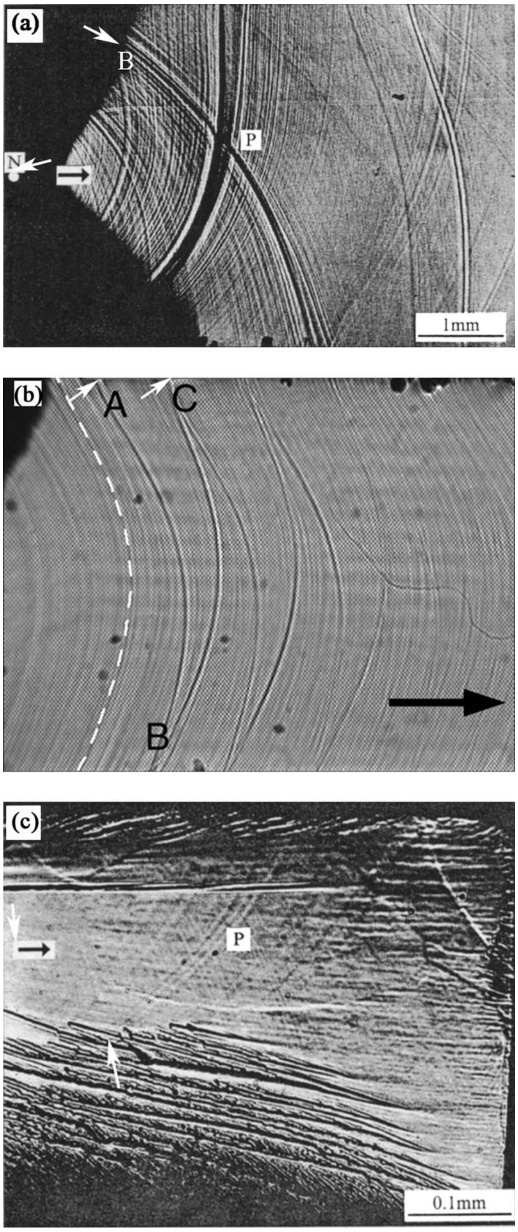

FIG. 3. (a) Fracture surface in inorganic glass loaded by uniaxial tension (modified from Ref. 8). Crack originated at *N* (shown by a white circle) and propagated in the direction shown by the black arrow. Crack velocity was analyzed at point *P* by the method shown in Fig. 1(b) [Eq. 1(b)]. White arrows: points of origin of the crack and the Wallner line, respectively, as calculated by our method. (b) Fracture surface in glass (modified from Ref. 1). Crack propagation direction is shown by a black arrow. The shear wave is excited by a special transducer and propagates in the vertical direction. White dashed line is an arrest mark or an undulation (Ref. 1). *AB* and *BC* depict Wallner lines. White arrows at *A* and *C*: the locations of the first Wallner lines origin and the end of the second one. (c) Fracture surface in a tungsten single crystal loaded by uniaxial tension (modified from Ref. 7). Crack velocity was calculated (Ref. 7) at point *P*. Black arrow shows crack propagation direction. White arrows: points of origin of the crack and the Wallner line, respectively, as calculated by our method.

ues were 474.21 and 474.33 m/s, respectively, as compared with 480 m/s of Ref. 1 The locations of the first Wallner lines origin and the end of the second one are shown by white arrows in Fig. 3(b). It can be argued that fracturing here is started at a line and not at a single point. In this case we denote the line origin as $y=0$ and the interaction at time t of the crack front and an acoustic wave emanating from $B(0,d)$ is given by

$$y = v_{\mathrm{cr}} t, \tag{8}$$

$$x^2 + (y-d)^2 = c_s^2 (t - t_1)^2. \tag{9}$$

Eliminating t from Eqs. (8) and (9) leads to Eq. (10), describing the Wallner line contour:

$$z \equiv x^2 + (y-d)^2 - w(y-s)^2 = 0. \tag{10}$$

Applying this approach to the calculation of Fig. 3(b) gives a value of 486.66 m/s, that is very similar to the ones obtained above by our "one point model" and by the authors of Ref. 1

The third example is shown in Fig. 3(c). The previously calculated[7] crack velocity ratio was 0.61, while our calculation yielded the value of 0.625. The locations of the crack and Wallner line origins evaluated by our method, unknown in Ref. 7, are shown by white arrows. The distance s that the crack propagates during the delay time t_1 here is 0.088 mm, while the distance between the crack and Wallner line origins is 0.128 mm. It means that the information here traveled at a velocity whose ratio to the transverse acoustic one is [by Eq. (7)] (0.128/0.088)0.63=0.916. This result is similar to the one obtained for the first experiment.

All three examples show that the velocity values obtained by our calculation are very similar to the ones procured in Refs. 1, 7, and 8. In comparison with the previous methods described above, our method is simpler, always easily implemented, and much more accurate. Let us reemphasize that for this method no accurate definition of crack or Wallner line origins is needed. Moreover, the coordinates of these points and the information velocity are obtained as a fringe benefit of the calculation.

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